Coaxial Cavities with Corrugated Inner Conductor for Gyrotrons

Christos T. Iatrou, Stefan Kern, and Alexander B. Pavelyev

Abstract*—This paper investigates coaxial gyrotron cavities with longitudinal slots on the inner conductor as a means to reduce the number of possible competing modes. In the analytic theory the corrugated surface is treated as a homogeneous impedance surface ("impedance corrugation") to obtain simple formulas for the characteristic equation of the eigenmodes, for the electromagnetic fields and the wall losses. The developed model applies if the number of slots is sufficiently high (cutoff wavelength much larger than the corrugation period). The characteristic equation in terms of the ratio C of the outer wall radius to the inner conductor radius is solved numerically to determine a range of eigenvalues and C where the eigenvalue curves are monotonically decreasing. In such a region a cavity having its inner conductor downtapered (radius decreasing toward the cavity output) can be used to reduce the diffractive quality factors of several modes, leaving the working mode undisturbed and without favoring other modes. In addition the electromagnetic field profiles are investigated, and in particular it is shown that for certain cavity parameters a mode could have its energy concentrated close to the inner conductor. As a check on the validity of the theoretical approximations, simulations with the MAFIA code are carried out. These give good agreement with the results of the analytic equations.

I. Introduction

HIGH-FREQUENCY, high-power gyrotron oscillators are under development for plasma heating in future fusion reactors. The main technological constraint in the design of a gyrotron cavity is the thermal wall loading [1], [2], which must be limited to 2–3 kW/cm^2 for long pulses or CW operation. To reduce the wall loading it is necessary to increase the cavity diameter, and thus high-frequency operation requires the use of high-order modes. As the mode spectrum becomes more dense for high-order modes, beam positioning does not provide a sufficient means for mode selection and stability, and mode competition becomes a serious physical constraint. Coaxial cavities have been proposed [3] to rarefy the mode spectrum and to reduce the competing action of neighboring modes, enabling stable single mode operation of the device.

In a coaxial geometry the eigenvalue χ_{mp} of a $\mathrm{TE}_{\mathrm{mp}}$ (or $\mathrm{TM}_{\mathrm{mp}}$) mode becomes a function of the ratio $C = R_o/R_i$ of the outer to the inner conductor radius. If either the outer wall or the coaxial insert is appropriately tapered, $\chi_{\mathrm{mp}}(z)$ is not constant, and a selective influence on the diffractive quality factor Q_{diff} of different modes becomes possible [3], [4]. This offers a means to change the importance of a mode concerning mode competition because the required starting current is proportional to $1/Q$. For high power gyrotron cavities the total Q-factor is mainly determined by the diffractive Q-factor. In the following discussion a gyrotron cavity is considered to consist of a weakly tapered waveguide with a total reflecting cutoff section in lower z-values and a partly reflective output in higher z-values. As the eigenvalue changes along z, the diffractive quality factor increases with positive $d\chi_{\mathrm{mp}}/dz$, compared to Q_{diff} in the corresponding hollow cavity, and decreases in the opposite case. This is due to the fact that with positive $d\chi_{\mathrm{mp}}/dz$ the outgoing wave (traveling in positive z-direction) travels toward increasing cutoff-frequency, thus toward decreasing energy velocity and so keeps more energy in the cavity than a wave with constant cutoff frequency. For example, with a downtapered (decreasing toward the cavity output) inner conductor, dC/dz is positive so that Q_{diff} increases with positive $d\chi_{\mathrm{mp}}/dC$ and vice versa. At a given parameter C, different modes will have different slopes $d\chi_{\mathrm{mp}}/dC$, and thus it is possible to increase their starting current requirements by decreasing their quality factors without changing the quality factor of the operating mode. On the other hand, this is not a general rule, and not all modes will move to the direction of a lower Q-factor. Some of them will increase their Q-factor because of a positive slope $d\chi_{\mathrm{mp}}/dC$ of the eigenvalue curve (in case of a downtapered rod), leading to serious mode competition problems.

To overcome this problem the introduction of longitudinal corrugations on the inner conductor surface is proposed. This results in a significant modification of the eigenvalue curves in terms of the parameter C. It will be shown that with appropriate selection of the slot depth, the eigenvalue curves $\chi_{\mathrm{mp}}(C)$ will exhibit a monotonic behavior with C, and therefore better handling of mode diffraction quality factors is possible.

The paper is organized as follows: In Section II, an approximate constant surface-impedance model is applied to the corrugated inner surface and the characteristic equation of such a configuration is obtained. The wall loading of the

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cavity is also considered. In Section III, numerical solutions of the analytic formulas are presented and the characteristic features of the system are discussed. The behavior of the electromagnetic field with respect to different parameters is presented and design principles for a coaxial gyrotron cavity are derived. In Section IV, the theoretical results are compared with those obtained from the finite integration technique code MAFIA. Section V is a summary and conclusion.

II. Impedance Model, Characteristic Equation, and Wall Loading

Consider a coaxial cavity with longitudinal corrugations on the wall of the inner conductor, as depicted in Fig. 1(a). The outer wall radius is denoted by R_o, the inner radius by R_i, and the depth of the slots by d. Region I refers to the space inside the slots, while region II is the area between the top surface of the corrugations and the outer wall. The treatment of such a problem should account for the azimuthal periodicity of the structure, and therefore spatial harmonics should be considered inside and outside the slots [5]–[7]. However, for a sufficiently large number N of slots, a much simpler approach based on an average surface impedance can be pursued [8]–[10]. Under the condition

$$s < \frac{\pi R_i}{m} \tag{1}$$

where $s = 2\pi R_i/N$ is the circumferential length of a period of corrugation and m is the number of field cyclic variations with ϕ (azimuthal index), the spatial harmonics are reduced to sufficiently small levels. Therefore, the field components of a $\mathrm{TE_{mp}}$ mode in region II are those of a usual coaxial resonator, given by

$$E_r = j\frac{m}{r}C_{\mathrm{mp}}Z_{\mathrm{mp}}(k_\perp r)e^{-jm\phi}V_{\max}\hat{f}(z) \tag{2a}$$

$$E_\phi = k_\perp C_{\mathrm{mp}}Z'_{\mathrm{mp}}(k_\perp r)e^{-jm\phi}V_{\max}\hat{f}(z) \tag{2b}$$

$$H_z = -j\frac{k_\perp^2}{k_0 Z_0}C_{\mathrm{mp}}Z_{\mathrm{mp}}(k_\perp r)e^{-jm\phi}V_{\max}\hat{f}(z) \tag{2c}$$

where the cylindrical function Z_{mp} is given by

$$Z_{\mathrm{mp}}(k_\perp r) = J_m(k_\perp r) + A_{\mathrm{mp}}Y_m(k_\perp r) \tag{3}$$

$J_m(x)$ and $Y_m(x)$ are Bessel and Neumann functions, with derivatives referring to their argument, $k_\perp = \chi_{\mathrm{mp}}/R_o$ is the transverse wavenumber, ω is the wave angular frequency, assuming a time dependence of $\exp(j\omega t)$, $k_0 = \omega/c$ is the free-space wavenumber, $Z_0 = (\mu_0/\varepsilon_0)^{-1/2}$ is the free-space wave impedance, $V_{\max}$ is the voltage that measures the maximum rms amplitude of the transverse electric field in the cavity, and $\hat{f}(z)$ is the field profile normalized to a maximum value of 1. The normalization constant C_{mp} will be determined later in (12). The transverse magnetic fields have been neglected as the gyrotron operates close to cutoff where these fields vanish.

Condition (1) ensures that the slot width l is smaller than $\lambda_c/2$ where λ_c is the cutoff wavelength, so that the fields can be assumed homogeneous inside the slots, although they can vary from one slot to the next according to the azimuthal wavenumber. Therefore, the z-component of the electric field vanishes inside the slot because it must obviously vanish at the ridges. This means that the surface $r = R_i$ behaves as a perfect electric conductor for TM modes. Consequently, for these modes the longitudinal slots have no effect on the fields and the system acts as a usual smooth-wall coaxial resonator. To show the effect on TE modes their characteristic equation is derived in the following by matching the average wall impedances of region I and region II.

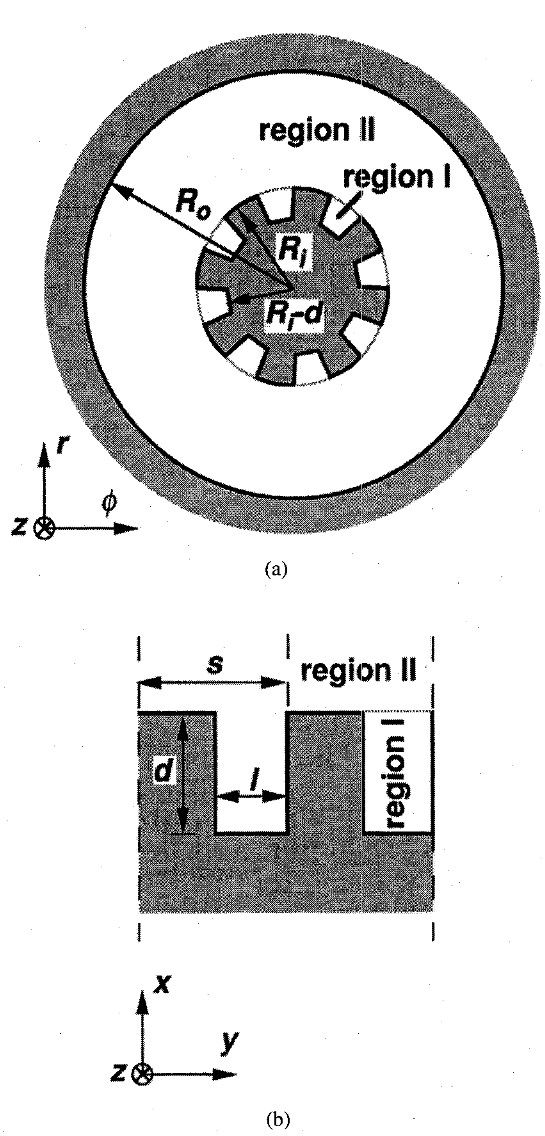


Fig. 1 (a) Cross section of the coaxial cavity. (b) Unfolded scheme of the corrugated rod.

To simplify the analysis further, the slot can be considered as part of a rectangular waveguide. In Fig. 1(b) the unfolded transverse structure and the coordinate system used in the analysis are presented. Inside a slot $0 \le x \le d$ (or equivalently $R_i - d \le x \le R_i$) the field should be y-independent and can be approximated by a part of a rectangular $\mathrm{TE}_{1,0}$ mode with field components

$$E_y = -k_\perp A_{10}\sin(k_\perp x)V_{\max}\hat{f}(z) \tag{4a}$$

$$H_z = -j\frac{k_\perp^2}{k_0 Z_0}A_{10}\cos(k_\perp x)V_{\max}\hat{f}(z). \tag{4b}$$

It is readily seen that the boundary condition $E_y = 0$ at $r = R_i - d$ is satisfied. The surface impedance at $r = R_i$ is anisotropic, with $Z_z^{\rm I} = E_z/H_x = 0$ and

$$Z_y^{\rm I} = (E_y/H_z)_{x=d} = \begin{cases} 0 & 0 \le y \le s - l \\ -jZ_0 \frac{k_0}{k_\perp} \tan(k_\perp d) & s - l < y < s \end{cases}. \tag{5}$$

From (5) one can compute an average surface impedance from region I to match it with that one calculated from region II. This average surface impedance is

$$\tilde{Z}_y^{\rm I} = \frac{1}{s} \int_0^s Z_y^{\rm I}(y)\, dy = -jZ_0 \frac{k_0}{k_\perp} w \tag{6}$$

where the normalized surface impedance w, as a function of the eigenvalue $\chi_{\rm mp}$ and the corrugation parameters, is given, by

$$w(\chi_{\rm mp}) = \frac{l}{s} \tan\left(\chi_{\rm mp} \frac{d}{R_o}\right). \tag{7}$$

Using the field components given in (2), which hold in region II, one computes the following surface impedance at $r = R_i$ from region II

$$Z_\phi^{\rm II} = (E_\varphi/H_z)_{r=R_i} = jZ_0 \frac{k_0}{k_\perp} \frac{Z'_{\rm mp}(k_\perp R_i)}{Z_{\rm mp}(k_\perp R_i)}. \tag{8}$$

Matching the two surface impedances (6) and (8), and applying the boundary condition for the tangential electric field component on the outer wall, we obtain the characteristic equation and the field coefficient $A_{\rm mp}$ for a coaxial geometry with axial impedance corrugations on the rod

$$J'_m(\chi_{\rm mp})[Y'_m(\chi_{\rm mp}/C) + wY_m(\chi_{\rm mp}/C)] - Y'_m(\chi_{\rm mp})[J'_m(\chi_{\rm mp}/C) + wJ_m(\chi_{\rm mp}/C)] = 0 \tag{9}$$

and

$$A_{\rm mp} = -\frac{J'_m(\chi_{\rm mp}/C) + wJ_m(\chi_{\rm mp}/C)}{Y'_m(\chi_{\rm mp}/C) + wY_m(\chi_{\rm mp}/C)} = -\frac{J'_m(\chi_{\rm mp})}{Y'_m(\chi_{\rm mp})}. \tag{10}$$

The rectangular-waveguide field coefficient A_{10} is computed by applying the continuity condition of the z-component of the magnetic field H_z at $r = R_i$

$$A_{10} = \frac{Z_{\rm mp}(\chi_{\rm mp}/C)}{\cos(\chi_{\rm mp} d/R_o)} C_{\rm mp} e^{-jm\phi_s} \tag{11}$$

where ϕ_s is the azimuthal angle of each slot. The normalization constant $C_{\rm mp}$ is obtained by applying the usual orthonormality condition [11] and it is given by

$$\frac{1}{C_{\rm mp}^2} = \pi(\chi_{\rm mp}^2 - m^2) Z_{\rm mp}^2(\chi_{\rm mp}) - \pi\left\{\frac{\chi_{\rm mp}^2}{C^2}(1+w^2) - \frac{\chi_{\rm mp}}{C} \times \left[w + \frac{\chi_{\rm mp} d}{R_o}\left(\frac{l}{s} + \frac{s}{l} w^2\right)\right] - m^2\right\} Z_{\rm mp}^2\left(\frac{\chi_{\rm mp}}{C}\right). \tag{12}$$

In the evaluation of the normalization constant $C_{\rm mp}$ the field energy over the whole space, including the slots, was considered.

The characteristic equation given by (9) is equivalent to that obtained by Li *et al.* [6] and Li and Li [7] in the limit of negligible spatial harmonics. They describe the field components inside the slots using cylindrical functions. In this case the following expression for an equivalent normalized surface impedance can be obtained

$$w = \left(\frac{N\theta}{\pi} \frac{\sin m\theta}{m\theta}\right) \times \frac{J_1(\chi_{\rm mp}/C) Y_1(\chi_{\rm mp}/C') - Y_1(\chi_{\rm mp}/C) J_1(\chi_{\rm mp}/C')}{J_0(\chi_{\rm mp}/C) Y_1(\chi_{\rm mp}/C') - Y_0(\chi_{\rm mp}/C) J_1(\chi_{\rm mp}/C')} \tag{13}$$

where N is the number of slots around the circumference, $2\theta = \pi/N$, and $C' = (R_i - d)/R_o$ characterizes the radial position of the bottom of the corrugations relative to the outer wall radius. Since they implicitly assume in their model $s = 2l$, the parenthesis in (13) becomes 1/2 in the limit of a large number of slots. In the next section it will be shown that the two ways of approaching the problem are almost equivalent.

The characteristic equation (9) can be readily applied in the limit of vanishing normalized surface impedance w to obtain the characteristic equation of a $\rm TE_{mp}$ mode in a coaxial cavity with smooth inner-conductor surface

$$J'_m(\chi_{\rm mp}/C) Y'_m(\chi_{\rm mp}) - Y'_m(\chi_{\rm mp}/C) J'_m(\chi_{\rm mp}) = 0. \tag{14}$$

The normalization factor $C_{\rm mp}$ and the field coefficient $A_{\rm mp}$ can be easily obtained from (12) and (10) under the same limiting condition $w = 0$. Note that the mode eigenvalues depend on the ratio C of the outer to the inner radius of the coaxial resonator. If the radius of the inner conductor is very small compared to the outer radius then the characteristic equation (14) becomes the usual characteristic equation of TE modes in an empty cylindrical resonator, that is $J'_m(\chi_{\rm mp}) = 0$. Modes with caustic radius $R_c = mR_o/\chi_{\rm mp}$ larger than the inner conductor radius are only slightly influenced by the presence of the rod. Solutions of (14) will be also presented in the next section.

Let us now calculate the ohmic losses of the microwave power on the resonator walls. The dissipated power density is given by [12]

$$\rho_{\rm ohm} = \frac{1}{2} R_s H_z H_z^* \tag{15}$$

where $R_s = 1/(\sigma\delta)$ is the surface resistance, σ is the conductivity of the material, and $\delta = [2/(\omega\mu\sigma)]^{1/2}$ the skin depth. The contribution of the transverse magnetic field components to the losses has been neglected since the gyrotron operates close to cutoff. To derive an expression of the wall loading in terms of the output power and the diffractive Q-factor of the resonator we use the energy balance equation to obtain

$$Q_{\rm diff} P_{\rm out} = \frac{1}{2} \varepsilon_0 \omega V_{\rm max}^2 \int_{-L/2}^{L/2} \hat{f}^2(z)\, dz \tag{16}$$

where L is the cavity length. Concerning gyrotron cavity design, the most important parameter is the peak value of the wall loading, which on the outer wall and on the top surface

of the corrugations is given by
Outer wall

$$\rho_o^{\text{peak}} = \frac{2\pi^2\delta}{\lambda^2}\frac{Z_{\text{mp}}^2(\chi_{\text{mp}})}{\int_{-L/2}^{L/2}\hat{f}^2(z)dz}C_{\text{mp}}^2 Q_{\text{diff}}P_{\text{out}}. \tag{17}$$

Top corrugation surface

$$\rho_{i,\text{top}}^{\text{peak}} = \frac{2\pi^2\delta}{\lambda^2}\frac{Z_{\text{mp}}^2(\chi_{\text{mp}}/C)}{\int_{-L/2}^{L/2}\hat{f}^2(z)dz}C_{\text{mp}}^2 Q_{\text{diff}}P_{\text{out}} \tag{18}$$

where the cylindrical function Z_{mp} is given in (3). In case of a Gaussian field profile $\hat{f}(z)$ the integral is approximately equal to $0.625L$.

The axial component H_z of the magnetic field inside the slot, given in (4b), induces wall currents on the radial surfaces of the corrugations, as well as at the bottom surface. The loading of these surfaces is readily found to be
radial surface

$$\rho_{i,\text{radial}}^{\text{peak}} = \frac{\rho_{i,\text{top}}^{\text{peak}}}{\cos^2(\chi_{\text{mp}}d/R_o)}\cos^2(\chi_{\text{mp}}x/R_o) \tag{19a}$$

bottom surface

$$\rho_{i,\text{bottom}}^{\text{peak}} = \frac{\rho_{i,\text{top}}^{\text{peak}}}{\cos^2(\chi_{\text{mp}}d/R_o)}. \tag{19b}$$

In the limit of vanishing cosine function these formulas still apply because the numerator also approaches zero there.

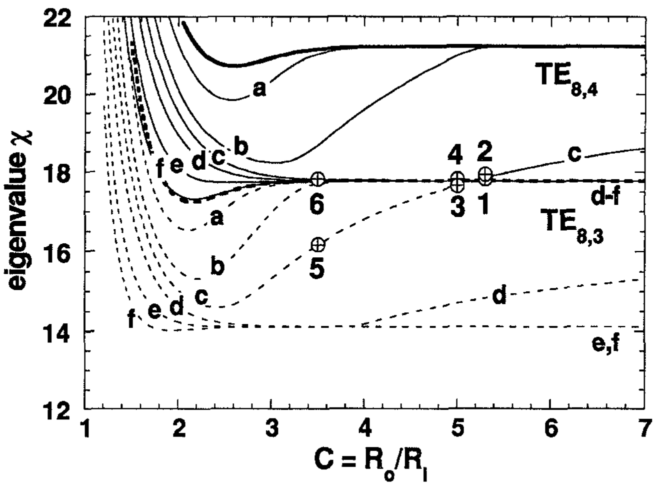
For a corrugation depth d close to a quarter of the free-space wavelength, which will be shown later to be an appropriate design value, the loading of the top surface of the corrugations ($r = R_i$) is negligible, as in this case the normalized surface impedance w tends to infinity and the RF magnetic field at R_i vanishes. Under these conditions the main part of the dissipated power on the inner conductor comes from the wall loading at the bottom, as well as at the radial surfaces of the corrugations.

III. Numerical Results and Discussion

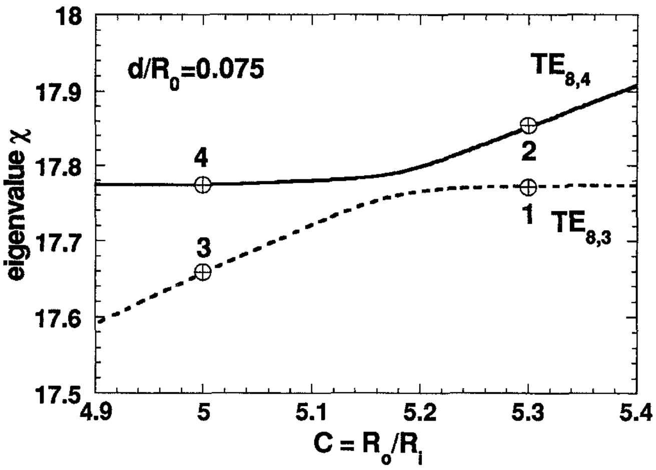
It was already mentioned that the main advantage of a coaxial geometry concerning mode competition is the dependence of the mode eigenvalue χ_{mp} on the radii-ratio parameter C. Typical solutions of the characteristic equation (9) as a function of C are shown in Fig. 2(a) with the corrugation depth d as parameter. The azimuthal index is set to $m = 8$ and the given examples are in a relatively low eigenvalue range in order to be able to compare with results obtained from numerical codes such as MAFIA. A realistic working mode for high power and high frequency gyrotrons will be of much higher order. The behavior of the eigenvalue curves $\chi_{\text{mp}}(C)$ will be described in terms of the ratio d/λ_c, where $\lambda_c = 2\pi R_o/\chi_{\text{mp}}$ is the cutoff wavelength. Since λ_c is a function of the eigenvalue χ, we choose the hollow waveguide eigenvalue of the $TE_{8,3}$ mode ($\chi_{8,3} = 17.774$) as a reference for the description of the parametric curves in Fig. 2(a). In the case of a smooth coaxial insert (thick curves in Fig. 2(a)) for large values of C (small R_i) the eigenvalues are practically independent of C and equal to the corresponding hollow waveguide eigenvalues. With decreasing C (increasing R_i), the influence of the rod becomes gradually more important. The eigenvalue curves exhibit first a positive slope $d\chi_{\text{mp}}/dC$ and after passing a minimal value they rapidly increase (negative slope $d\chi_{\text{mp}}/dC$) in the region of strong influence of the rod (small C). When the corrugation is added, the region of positive slope broadens and moves toward higher C values (curves a, b, and c). The eigenvalue minimum decreases and also occurs at higher C values. As the corrugation depth approaches $\lambda_c/4$, the minimum value of an eigenvalue curve becomes equal to the eigenvalue of the next lower radial mode with the same azimuthal index m (solid curve c or dashed curve d). The eigenvalue curve now consists of a constant part at the corresponding hollow waveguide eigenvalue (not visible in Fig. 2(a), see thick solid line in Fig. 3), then a part having positive slope in relatively high C values and another nearly constant part at a value which is equal to the hollow waveguide eigenvalue of the next lower radial mode. At that point where the influence of a smooth rod started, indicated by a decrease in eigenvalue, the eigenvalue in the corrugated system starts to increase, so that the rest of the curve (toward smaller C) exhibits only a negative slope $d\chi_{\text{mp}}/dC$. For $\lambda_c/4 < d < 3\lambda_c/8$ (solid curves d and e in Fig. 2(a) or curves above $\chi \approx 47$ in Fig. 3) the positive slope vanishes and the eigenvalue curves remain with a constant undisturbed part, and a part of negative slope due to a strong influence of the rod. For higher d the eigenvalue curves exhibit a positive slope again, until $d \approx \lambda_c/2$, where they become approximately the same with those of a smooth coaxial cavity (solid curve f in Fig. 2(a)). In even higher d the described behavior will be repeated periodically with a period of $\lambda_c/2$.

It has been shown in [3] and [4] that in a coaxial cavity with tapered smooth inner conductor there is the possibility of selective influence on the diffractive quality factor of different modes, and thus on the starting current requirements of these modes. In a coaxial cavity with a downtapered rod a positive slope $d\chi_{\text{mp}}/dC$ of the eigenvalue curve leads to an increase of the mode diffractive quality factor, and vice versa. Nevertheless, it is obvious from the curves in Fig. 2(a), that in case of a smooth rod (thick solid curves) some modes will have an increase of their Q-factor while some others will have a decrease of Q, depending on the slope of their eigenvalue curve at the design range of the C-parameter. It will be shown later that to avoid high wall loading on the rod the working mode must remain nearly undisturbed ($d\chi_{\text{mp}}/dC \approx 0$), which sets a limit on the minimal usable C value. This means also that the quality factor of the working mode will remain the same as in the corresponding hollow cavity. Therefore any increase in quality factors of possible competitors must be avoided.

Modes with a caustic radius close to that of the working mode are usually the most serious competitors because they couple as well to the electron beam as the working mode. The influence of the inner conductor on the eigenvalues starts approximately when the rod gets near to the caustic radius of a mode, so that obviously only the modes with smaller caustic radius can be disturbed by the rod if C is chosen as



(a)



(b)

Fig. 2. (a) Eigenvalue versus C for $TE_{8,3}$ mode (dashed lines) and $TE_{8,4}$ mode (solid lines). Thick lines are for smooth inner conductor ($d = 0$), thin lines as labeled: $d/\lambda_c = 0.08$ (a), $d/\lambda_c = 0.17$ (b), $d/\lambda_c = 0.21$ (c), $d/\lambda_c = 0.26$ (d), $d/\lambda_c = 0.38$ (e), $d/\lambda_c = 0.50$ (f) with $\lambda_c/R_o = 0.35$ and $l/s = 0.5$. (b) Section from (a) around the "degeneracy point" between $TE_{8,3}$ and $TE_{8,4}$ (only curves (c) with $d/\lambda_c = 0.21$, $d/R_o = 0.075$).

small as acceptable. But in case of a smooth inner conductor these weakly influenced modes will unfortunately exhibit a positive slope in their eigenvalue curves, as discussed above, which will lead to an increased Q-factor and possibly to unstable operation, if a downtapered rod is used. Employing an uptapered rod would solve the problem only for the few weakly influenced modes, but would increase the quality factors of all the strongly influenced modes. Actually the problem is that the eigenvalue curves for a coaxial geometry with smooth rod are not monotonic.

On the other hand, it is shown in the first paragraph of this Section that the introduction of longitudinal corrugations on the inner conductor solves this problem in the sense that with a corrugation depth in the range $\lambda_c/4 < d < 3\lambda_c/8$ the eigenvalue curves $\chi_{\mathrm{mp}}(C)$ are monotonic with a negative or vanishing slope for all values of C. Therefore by choosing the minimal acceptable value for C, imposed by wall loading, the working mode and the other noninfluenced modes will keep the same quality factor as in a hollow cavity, while the quality factors of some of the competing modes will be decreased. Thus a first design rule for coaxial gyrotron cavities could be to use a downtapered rod and a corrugation with slot depth slightly larger than $\lambda_c/4$. Nevertheless, to avoid mode competition with higher cyclotron harmonics, it is desirable to keep the slot depth as small as possible. If, for example, $d = \lambda_c/4$ at the first cyclotron harmonic, the modes at the second cyclotron harmonic, with eigenvalues twice as high, will not be influenced by the corrugation since the slot depth corresponds to $d = \lambda_c/2$ in their eigenvalue region. Thus some of them will exhibit a positive slope and may become serious competitors due to an increased quality factor. To avoid positive slopes in eigenvalue curves at the second cyclotron harmonic the slot depth must be slightly decreased from $d = \lambda_c/4$, where λ_c is the cutoff wavelength of the operating mode.

In the range $d < \lambda_c/4$, a slot depth d can be found such that for a restricted design range of C and χ the curves $\chi_{\mathrm{mp}}(C)$ show only a negative slope, even though at higher C values, out of the design range, a positive slope appears. This is actually demonstrated in Fig. 3, which gives an overview of the eigenvalue curves of $m = 8$ modes at $d/R_o = 0.033$. At an eigenvalue $\chi \approx 47$ the chosen corrugation depth corresponds to $\lambda_c/4$. It is seen that the positive slope parts seem to form a single, additional "eigenvalue curve" as $\lambda_c/4$ approaches d and for high C values. This curve can be approximated by the solution of the following equation

$$Y_m'(\chi_{\mathrm{mp}}/C) + wY_m(\chi_{\mathrm{mp}}/C) = 0. \qquad (20)$$

This equation is derived from (9) by dividing by $J_m'(\chi_{\mathrm{mp}})$ and neglecting the Bessel function $J_m(\chi_{\mathrm{mp}}/C)$ and its derivative, which are small compared to the Neumann function $Y_m(\chi_{\mathrm{mp}}/C)$ for small arguments. The actual design problem is now to determine a slot depth d smaller than $\lambda_c/4$ for the working mode such that no positive slope in the eigenvalue curves of the working mode and any possible competing mode with different azimuthal index can appear inside the region of interest, which is given by the maximum C of the cavity and the cyclotron amplification bandwidth. It was already mentioned that modes with caustic radius smaller than the caustic of the working mode will exhibit a negative slope in a corrugated system with a slot depth approaching $\lambda_c/4$. In the narrow eigenvalue range, which corresponds to the cyclotron amplification bandwidth (±5%) these are practically all the modes with $m < m_0$ where m_0 is the azimuthal index of the operating mode. Since the caustic radius of the operating mode must be greater than the inner conductor radius, the inequality $\chi/m < C_0$ holds for $m \geq m_0$ in the cyclotron amplification bandwidth. Therefore, (20) can be further simplified using the recurrence relations for Neumann functions to yield

$$\chi_{\mathrm{mp}} = \frac{mC}{w(\chi_{\mathrm{mp}})} \qquad (21)$$

where the term $Y_m'(\chi_{\mathrm{mp}}/C)/Y_m(\chi_{\mathrm{mp}}/C)$ has been neglected, which is an acceptable approximation for $C > \chi_{\mathrm{mp}}/m$. Equation (21) is an approximation of the undesired positive slope parts of the eigenvalue curves, so that for a given C

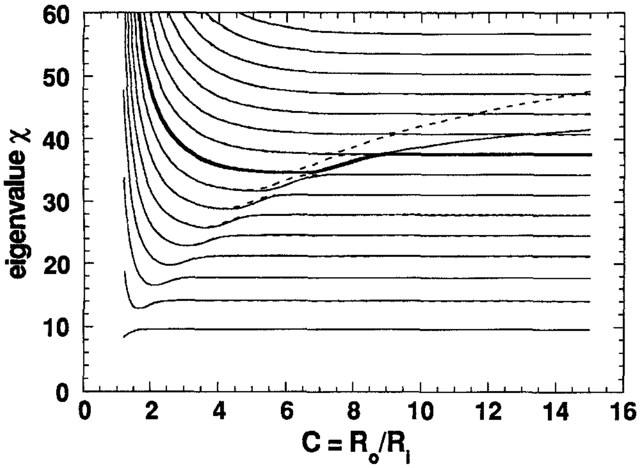


Fig. 3. Eigenvalue versus C for modes with $m = 8$ at $d/R_o = 0.033$ and $l/s = 0.5$. Solid lines are solutions of (9) taking w from (7), dashed lines are also solutions of (9), but taking w from (13). The thick solid line is the eigenvalue curve of $TE_{8,9}$.

and m, any curve of greater eigenvalues will not exhibit a positive slope. Therefore for a given minimum eigenvalue a minimum usable d in terms of λ_c can be determined using (7). The azimuthal index of the possible competitors is considered to be smaller than $2m_0$. With $l/s = 1/2$ a minimum d of $0.21\lambda_c$ ($w \approx 2$) is computed. This slot depth results in $d \approx 0.4\lambda_c$ at the second cyclotron harmonic, so that only small positive slopes in the eigenvalue curves can appear there. The minimum d can be slightly decreased if l/s is increased up to the mechanically feasible limit.

Let us now summarize the derived design principles of a coaxial gyrotron cavity with corrugated rod. The inner conductor should be downtapered (decreasing radius toward the cavity output), with radius R_i as large as possible to influence a maximum number of modes. The working mode should remain undisturbed, which sets an upper limit to the radius and a corresponding lower limit to C. An effective slot depth of $d \approx 0.2\lambda_c$ is considered as a proper design value to improve the stability of the operating mode against competitors in the first and second cyclotron harmonic. The effective corrugation depth d can be increased up to $3\lambda_c/8$, if mode competition with the second cyclotron harmonic is not considered.

To compare the presented model to that of Li *et al.* [6], solutions of the characteristic equation (9) are given in Fig. 3 using expression (13) for the normalized impedance w for the same geometrical slot depth d (dashed curves, mostly covered by the solid curves). It can be seen that the behavior is similar in both approaches. Both solutions can be made to coincide nearly exactly, if an effective d is used in (13), in this special case at $d/R_o = 0.04$. The difference in effective slot depth is explained by the different geometry of the two models, since Li *et al.* [6] use a nonrectangular slot geometry with boundary planes along the cylindrical coordinates. In general one can apply (7) to corrugations with nonrectangular shaped slots, if an effective slot depth is used [13]. This also means that one has to take care and use an effective slot depth in case the slots are not perfectly rectangular.

To check the applicability of corrugated cavities to gyrotrons, the electromagnetic fields of the modes must be investigated. This also will give an insight into the behavior of the positive-slope parts of the eigenvalue curves, which seem to form a single, additional curve for high C values, as discussed above. In Fig. 4 the radial profile of the H_z component is presented at six different points marked in Fig. 2. Note that the points 1, 3, and 5 belong to the same eigenvalue curve, while 2, 4, and 6 belong to the next higher radial mode. These two curves actually do not touch, although they are approaching very close to each other (see Fig. 2(b)). From Fig. 4 one can recognize two kinds of fields. In the parts of the eigenvalue curves with positive slope the energy is concentrated near the rod, while in the constant parts the field profile is similar to the commonly known one of the hollow waveguide, and is therefore well suited to be used in a gyrotron cavity. The fields at points 1, 4, and 6 show the character of a $TE_{8,3}$ mode, even though points 4 and 6 belong to the fourth eigenvalue curve. The fields at points 2, 3, and 5, with their maximum on the inner surface, are permitted by the average surface impedance at $r = R_i$. For convenience we will call a mode with such a field pattern "inner mode" in the following. The inner mode appears in all positive-slope regions of the eigenvalue curves, if the slot depth is sufficiently high. As it is concentrated on the rod, it is influenced even at very high C values, in contrast to the usual modes. Therefore the inner mode looks like an independent additional mode for high C values, where the usual modes are not disturbed by the rod. Interaction between the inner mode and modes with the usual transverse structure takes place only at the "points of degeneracy," which indeed are not crossing points (see Fig. 2(b)). As it can be seen in Fig. 4 the field at the inner boundary $r = R_i$ changes sign at these points. This indicates that the "eigenvalue curve" of the inner mode, as given by (20), is not continuous but consists of the positive-slope parts of the different eigenvalue curves. It should be pointed out again that this mode does not exist for $\lambda_c/4 < d < \lambda_c/2$.

To give a typical example for wall losses the loading of the cavity walls has been computed using (17)–(19) for a $TE_{8,3}$ mode. The frequency was assumed to be 28 GHz, the output power 1 MW and the diffractive quality factor 1200, while the conductivity was that of ideal copper. The corrugation depth was taken $d/R_o = 0.075$. In Fig. 5 the wall loading on the outer wall (thick line) and on the bottom surface of the corrugations (dashed line) is presented together with the eigenvalue curve (thin line). The part of the eigenvalue curve with positive slope is included to study the influence of the inner mode on wall loading. The losses on the top surface of the corrugation are about a factor of 20 lower than those on the bottom. For $C > 5$ the inner mode causes a strong loading on the bottom surface of the corrugations while the outer wall is nearly unloaded. Note that with such a high loading the decreased ohmic quality factor could dominate the increased diffractive quality factor, thus increasing the starting current of the inner mode. In the constant part of the eigenvalue curve ($3.5 < C < 5$), where the mode is nearly an undisturbed $TE_{8,3}$, the loading of the outer wall is approximately constant to 0.7 kW/cm^2 while that of the bottom surface is decreased

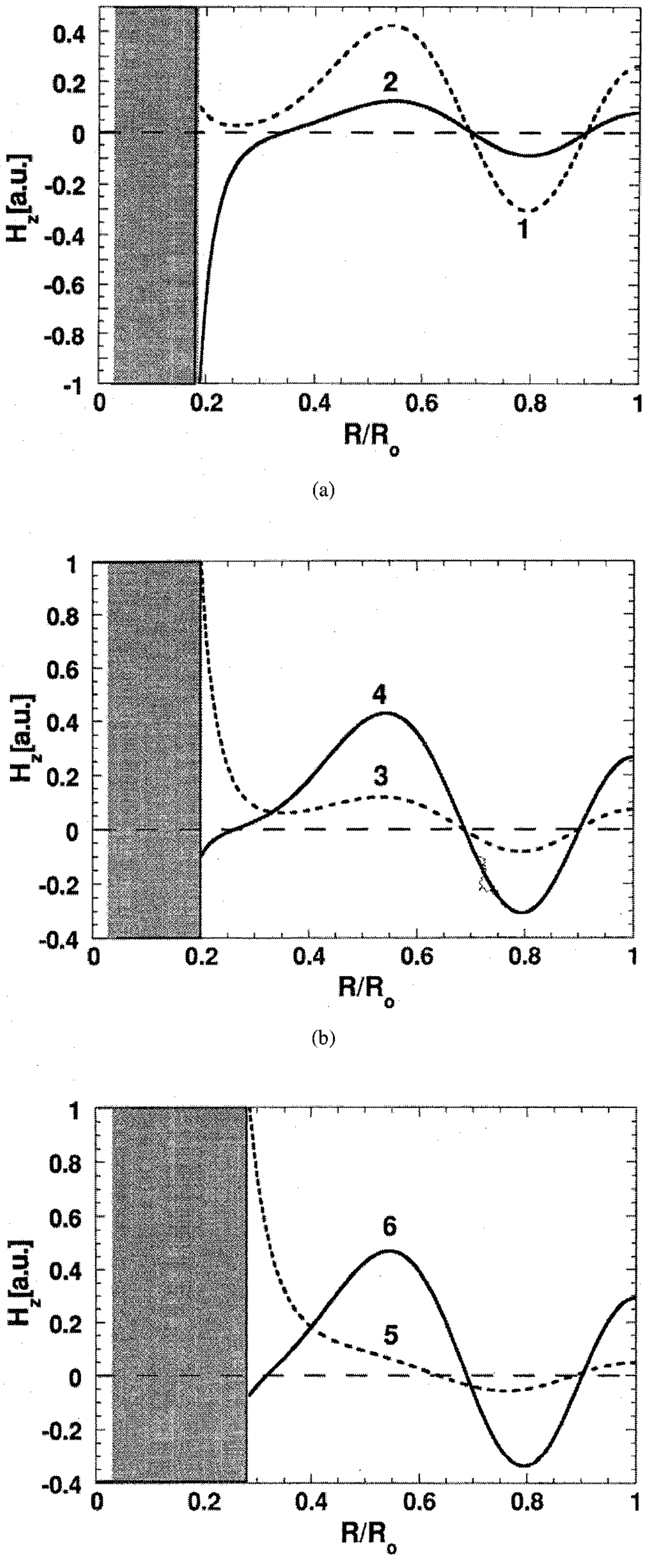


Fig. 4. (a) Radial profile of the H_z-component of $TE_{8,3}$ (1) and the inner mode (2) at $C = 5.3$ ($d/R_o = 0.075$). Curve numbers refer to the marked points in Fig. 2(a) or (b). (b) Radial profile of the H_z-component of $TE_{8,4}$ (4) and the inner mode (3) at $C = 5.0$ ($d/R_o = 0.075$). Curve numbers refer to the marked points in Fig. 2(a) or (b). Note that the $TE_{8,4}$ now shows the field structure of a $TE_{8,3}$ mode. (c) Radial profile of the H_z-component of $TE_{8,4}$ (6) and the inner mode (5) at $C = 3.5$ ($d/R_o = 0.075$). Curve numbers refer to the marked points in Fig. 2(a). Note that the $TE_{8,4}$ now shows the field structure of a $TE_{8,3}$ mode.

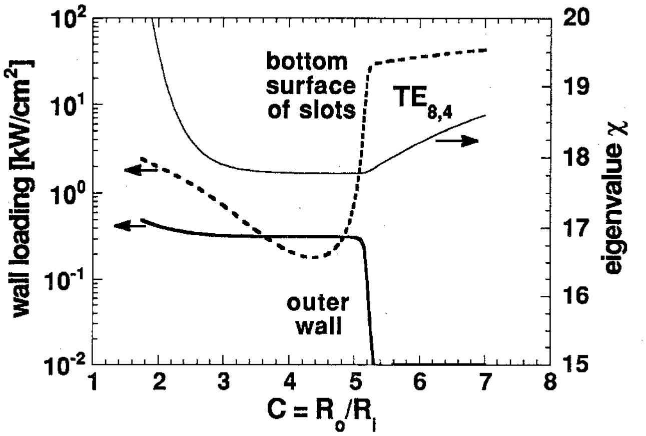


Fig. 5. Wall loading versus C for $TE_{8,4}$ ($d/R_o = 0.075$). The inner mode appears for $C > 5$.

to an acceptable level of 0.3 to 1 kW/cm^2. By further decrease of C the loading of both surfaces increases, especially the one at the bottom surface. Therefore, with regard to wall loading the design range of the C parameter is limited in general, in this example to $3.5 < C < 5$. In this range the wall loading at the bottom of the corrugated surface is of the same order as in the case of smooth inner conductor.

IV. Comparison with MAFIA Code

MAFIA is a numerical code using the finite integration technique (FIT) algorithm to solve Maxwell's equations [14]. In this study MAFIA has been used to compare with the results of the theoretical model presented in Section II. Two-dimensional simulations were performed in a sector of $\pi/8$ of a coaxial waveguide with five longitudinal slots ($l/s = 1/2$). Assuming ideal electric conductors on the radial boundary of this sector the eigenmodes in such a geometry are $TE_{8(n-1),p}$ and $TM_{8n,p}$ ($n \in N$). A number of simulations were carried out with different ratio C and corrugation depth d. As an example, the eigenvalues found by MAFIA for $d/R_o = 0.075$ are compared in Fig. 6 with the solutions of (9). In any case, the eigenvalues determined by both methods were equal up to the accuracy of the MAFIA code, which was limited by discretisation. Note that in Fig. 6 also the increasing positive slope for disturbed modes is found.

In Fig. 7(a) and (b) the radial profile of the H_z magnetic field component evaluated by MAFIA (thick lines) and by expressions (2c) and (4b) (thin lines) is presented, assuming $C = 2$, and $d/R_o = 0.075$. Excellent agreement is found both inside the slots, where the field is approximated by a trigonometric function, and outside the slots, where the field is expressed in terms of Bessel functions. Modes with the field concentrated on the inner conductor are also indicated by MAFIA simulations, as can be seen in Fig. 7(b).

Good agreement was found concerning wall loading computed from MAFIA simulations and compared to the corresponding results from (17)–(19), especially for the ratio between losses at the top surface of the corrugations to the bottom surface.

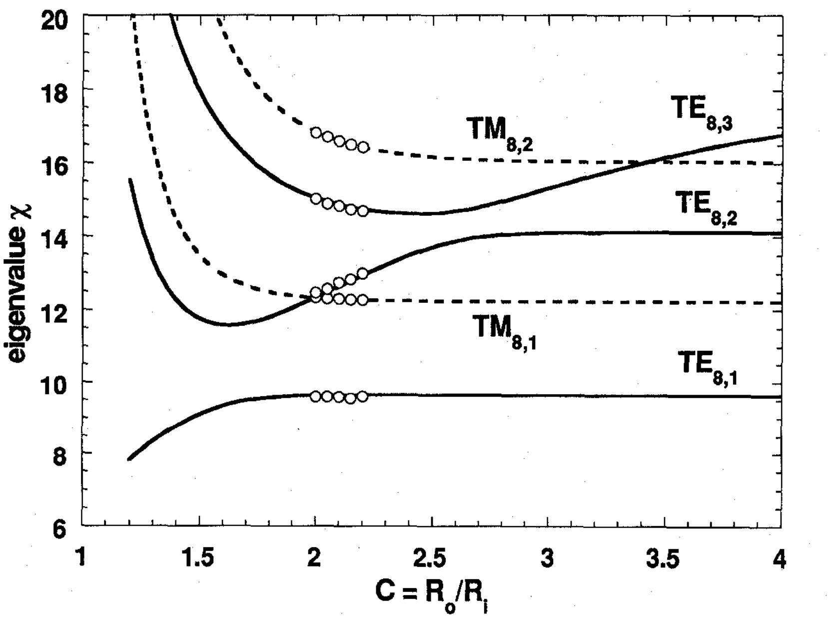


Fig. 6. Comparison of eigenvalues curves between MAFIA (circles) and theory (d/R_o = 0.075 and l/s = 0.5).

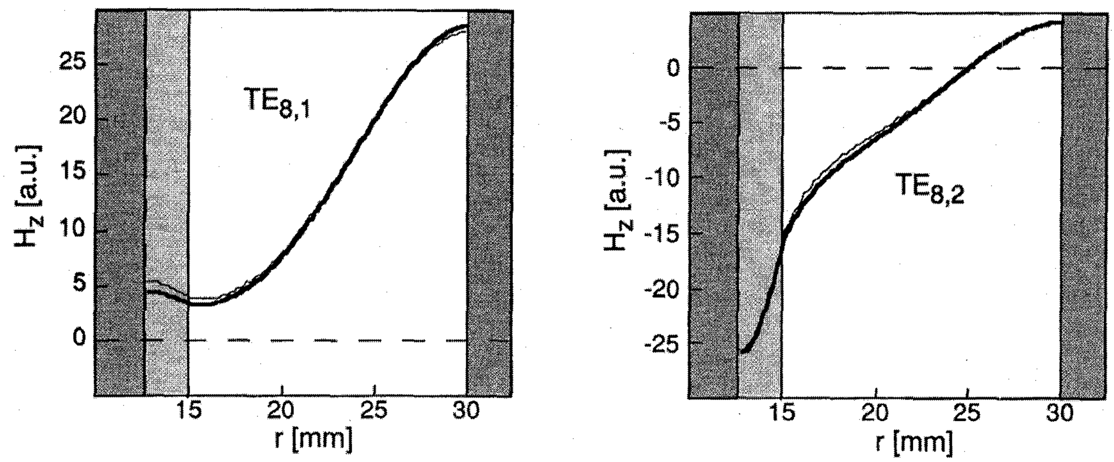


Fig. 7. Radial profile of the H_z-component of $TE_{8,1}$ and $TE_{8,2}$ from MAFIA (thick) and theory ($C = 2, d/R_o = 0.075$).

V. Conclusion

A coaxial cavity configuration with longitudinal corrugations on the inner conductor has been investigated in the case of small corrugation periodicity (constant impedance surface). This geometry is proposed for use in highly overmoded cavities of high power gyrotrons in order to reduce the problem of mode competition. With an appropriate slot depth ($d \approx 0.2\lambda_c - 3\lambda_c/8$), the corrugations remove any positive slope $d\chi_{\rm mp}/dC$ from the eigenvalue curves $\chi_{\rm mp}(C)$ for all the modes inside the cyclotron amplification bandwidth, so that no unwanted modes with increased quality factor can appear in the spectrum of a coaxial cavity with downtapered inner conductor. It is therefore possible to design coaxial cavities, where only the chosen working mode $TE_{\rm mp}$ and modes with the same or higher caustic radius $R_c = mR_o/\chi_{\rm mp}$ exhibit a high diffractive quality factor, while $Q_{\rm diff}$ of the other modes is considerably reduced. Since the required starting current is proportional to $1/Q$, modes with decreased $Q_{\rm diff}$ require an increased beam current to start oscillations in the gyrotron cavity. Therefore, compared to a coaxial cavity with smooth rod, modes with lower caustic radius are no longer serious competitors. On the other hand, modes with caustic radius considerably higher than that of the working mode couple weakly to the electron beam, because they exhibit low fields at the beam position. Finally, in a cavity with properly corrugated and downtapered inner conductor only modes with caustic radius close to the caustic radius of the working mode remain as potentially dangerous competitors. Within the cyclotron amplification bandwidth these are the modes with azimuthal index m close to the azimuthal index of the working mode. A quality-factor spectrum of a coaxial cavity with corrugated and downtapered inner conductor and $d \approx 0.22\lambda_c$ was already given in [15].

An impedance corrugated rod with $d < \lambda_c/4$ permits the existence of a type of mode (inner mode) with its field concentrated on the inner conductor. For low C the usual waveguide modes transform to the inner mode along the eigenvalue curve, where $d\chi_{\mathrm{mp}}/dC$ is positive, and they end up at the eigenvalue of the neighboring radial mode with the same m. For high C the inner mode behaves as an additional mode with an eigenvalue approximately described by (20) or (21). This mode does not interfere with the usual modes, except for some points where it is degenerate with them. One possible problem caused by this inner mode could be increased mode conversion in the resonator uptaper section, where C rises from the cavity C_0 to infinity. Therefore the corrugations should be stopped before C becomes high enough to permit the appearance of such a mode.

Care has to be taken to avoid mode competition between the cyclotron harmonics due to a possible increase of the quality factor at the second cyclotron harmonic. A corrugation depth equal to $\lambda_c/4$ for the operating mode (first cyclotron harmonic) results in an effective depth of $\lambda_c/2$ in the second cyclotron harmonic eigenvalue region. Then the positive slope of the eigenvalue curves appears as in the case of smooth inner conductor, and the Q-factor of these modes increases for a downtapered inner conductor. These modes could become unexpected competitors, as their starting currents can be comparable to the starting current of the working mode. The problem is solved by using a slightly smaller slot depth of $d \approx 0.2\lambda_c$. On the other hand, employing $d \geq \lambda_c/4$ could be a means to enforce oscillations at the second cyclotron harmonic in low power gyrotrons. Note that for these modes wall losses on the rod will be much higher than at the outer wall, so that the gyrotron output power must be limited to a few kW.

The Forschungszentrum Karlsruhe and the IAP Nizhny Novgorod will investigate the presented design principles in a joint experiment [15]. A $TE_{28,16}$ coaxial gyrotron with 1.5 MW output power at 140 GHz is already under construction, and first experiments are expected in autumn 1995.

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